Real Numbers

chapter 1

Set:- A Collection of well defined and distinct objects is called a set.

Set (Collection of first five Natural numbers)

$\{1, 2, 3, 4, 5\}$

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Collection of any five Natural numbers is NOT a set.

Natural Numbers:- $\mathbb{N} = \{1, 2, 3, \ldots\}$

Whole Numbers:- $W = \{0, 1, 2, 3, \ldots\}$

Even Numbers:- $E = \{0, \pm 2, \pm 4, \pm 6, \ldots\}$

$= \{\ldots, -6, -4, -2, 0, 2, 4, 6, \ldots\}$

Odd Numbers $= O = \{1, 3, 5, 7, \ldots\}$

$= \{2n+1 : n \in \mathbb{Z}\}$

Integers $= \mathbb{I} = \mathbb{Z} = \{0, \pm 1, \pm 2, \pm 3, \ldots\}$

$= \{\pm n : n \in \mathbb{W}\}$

$= \{\ldots, -3, -2, -1, \underbrace{0, 1, 2, 3, \ldots}_{\text{whole numbers}}\}$

Non-negative integers

Prime Numbers $= \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29, \ldots\}$

The Natural numbers are not prime and called Composite Numbers. 1 is neither prime nor Composite

Rational Numbers:- $Q = \left\{ \frac{p}{q}, p \in Z, q \in Z, q \neq 0 \right.$
$\left. \frac{p}{q} \text{ must be in its lowest form} \right\}$

$\frac{9^{1}}{27_{3}} = \frac{1}{3}$ $p \in Z$ $q \in Z$ $q \neq 0$
$1 \in Z$ $3 \in Z$ $3 \neq 0$

$\frac{\sqrt{4}}{16} = \frac{2}{16} = \frac{1}{8}$ $1 \in Z, 8 \in Z$ $\frac{3}{\sqrt{16}} = \frac{3}{4}$

$\{\sqrt{0}, \sqrt{1}, \sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}, \sqrt{9}, \sqrt{10}, \sqrt{11} \ldots\}$

$= \{0, 1, \sqrt{2}, \sqrt{3}, 2, \sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}, 3, \sqrt{10}, \sqrt{11}, \ldots\}$

The Numbers which are not rational are called irrational numbers. $\frac{\sqrt{3}}{16}$

Real Numbers = $R = QUQ'$

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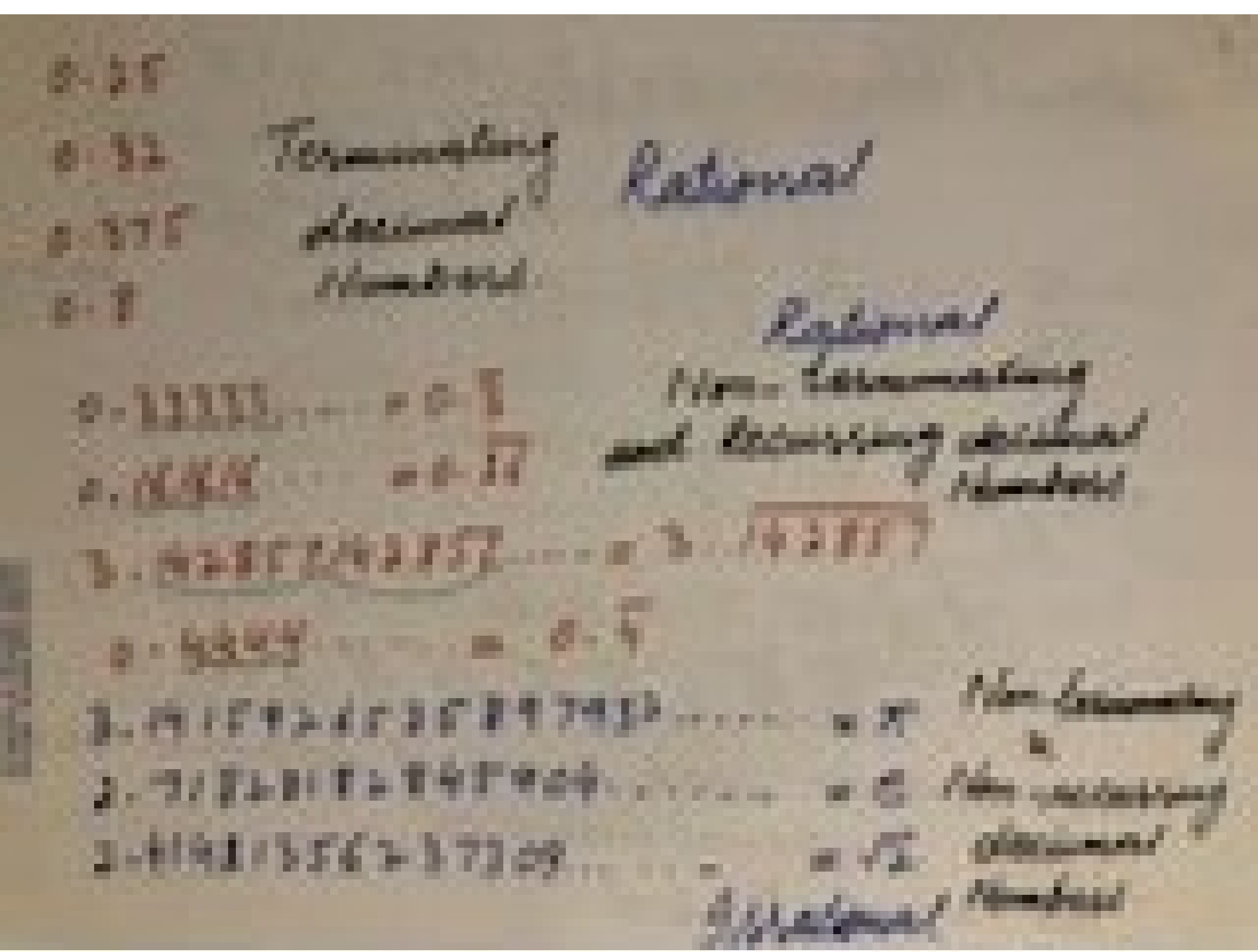

Example 1: Identify the following decimal numbers as rational or irrational numbers.

0.35 Rational (Terminating decimal number).

0.444.... Rational (Non-terminating and recurring decimal number).

3.36788542 ..... Irrational (Non-terminating and Non-recurring decimal number)

$3.\overline{5} = 3.555....$ Rational (Non-terminating and recurring decimal number).

1.7099775947..... Irrational Non-terminating and Non-recurring decimal number.

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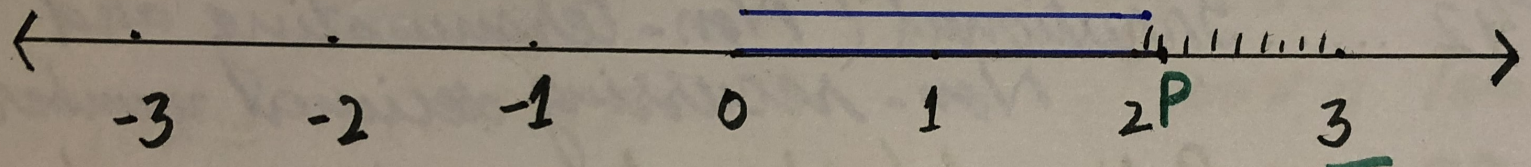
Example 2:- Represent $\sqrt{5}$ on number line.

$$\sqrt{5} = 2.236.....$$

$$\sqrt{5} = 2.2$$



$$|OP| = 2.2 \approx \sqrt{5}$$

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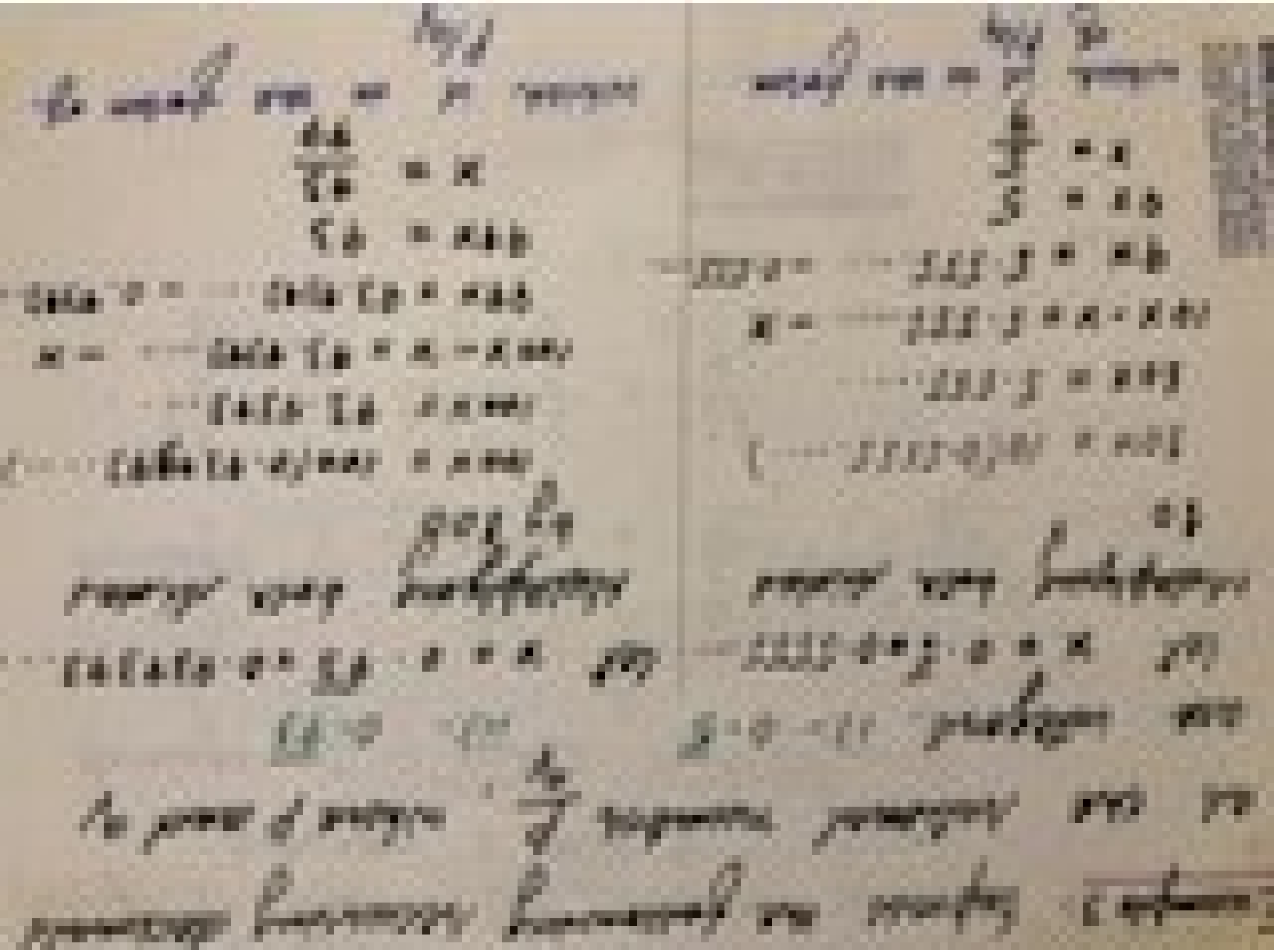
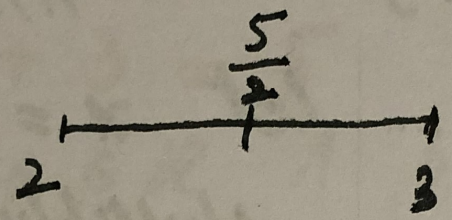


Example 4:- Insert two rational numbers between 2 and 3.

Average of 2 and 3 $= \frac{2+3}{2} = \frac{5}{2}$



Average of $\frac{5}{2}$ and 3 $= \frac{\frac{5}{2}+3}{2} = \frac{\frac{5+6}{2}}{2}$

$= \frac{\frac{11}{2}}{2} = \frac{11}{2} \times \frac{1}{2} = \frac{11}{4}$

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Properties of Real numbers

Addition Properties

Closure : $a + b = c$

$\forall\ a, b \in \mathbb{R}$ $\quad a + b \in \mathbb{R}$

Ex: $\quad 2 + 3 \in \mathbb{R}$

Commutative : $a + b = b + a$

$\forall\ a, b \in \mathbb{R}$ $\quad a + b = b + a$

Ex: $\quad 2 + 3 = 3 + 2$

Associative:- $1+(2+3) = (1+2)+3$

$1+5 = 3+3$

$6 = 6$

If $a \in \mathbb{R}$, $b \in \mathbb{R}$ & $c \in \mathbb{R}$

then $a+(b+c) = (a+b)+c$

Identity:- 0 is Additive identity

$5+0 = 5 = 0+5$

If $a \in \mathbb{R}$, then $a+0 = a = 0+a$

Inverse:- $5+(-5) = 0 = (-5)+5$

If $a \in \mathbb{R}$ then $a+(-a) = 0 = (-a)+a$

If $a+b = 0$, then a and b are additive inverses of each other.

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Multiplicative Inverse

Definition: [illegible]

Property [illegible]

Commutative Property: [illegible]

Distributive Property: [illegible]

Associative Property of Multiplication: [illegible]

Identity:- 1 is multiplicative identity

$$5 \times 1 = 5 = 1 \times 5$$

If $a \in \mathbb{R}$, then $a \times 1 = a = 1 \times a$

Inverse:- $5 \times \frac{1}{5} = 1 = \frac{1}{5} \times 5$

If $a \in \mathbb{R}$, then $a \times \frac{1}{a} = 1 = \frac{1}{a} \times a$

If $ab = 1$, then a and b are said to be multiplicative inverses of each other.

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Distributive property

Left distributive property of multiplication over addition

If a, b, c are real numbers, then $a(b+c) = ab + ac$

Left distributive property of multiplication over subtraction

If a, b, c are real numbers, then $a(b-c) = ab - ac$

$(3+4)2 = 3(2) + 4(2)$

If a, b, c are real numbers,

then $(a+b)c = ac + bc$

Right distributive law of multiplication over addition

$(4-3)2 = 4(2) - 3(2)$

If a, b, c are real numbers,

then $(a-b)c = ac - bc$

Right distributive law of multiplication over subtraction.

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Properties of equality of real numbers :-

Reflexive property :- $2 = 2$

For all $a \in R$, $a = a$

$\forall$

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Symmetric property :- $\frac{2}{4} = \frac{1}{2}$

$\forall a, b \in R$, if $a = b$ then $b = a$

$a = b \implies b = a$

Transitive property :- $\frac{2}{4} = \frac{1}{2}$, $\frac{1}{2} = \frac{3}{6}$

$\implies \frac{2}{4} = \frac{3}{6}$

$\forall a, b, c \in R$ if $a = b \wedge b = c$, then $a = c$

Additive property :- If $\frac{2}{4} = \frac{1}{2}$, then $\frac{2}{4} + 1 = \frac{1}{2} + 1$

$\forall\ a, b, c \in R$ If $a = b$, then $a + c = b + c$

Multiplicative property If $\frac{2}{4} = \frac{1}{2}$, then $\frac{2}{4} \times 2 = \frac{1}{2} \times 2$

$\forall\ a, b, c \in R$, If $a = b$, then $ac = bc$

Cancellation property w.r.t addition :- If $\frac{2}{4} + 1 = \frac{1}{2} + 1$, then $\frac{2}{4} = \frac{1}{2}$

$\forall\ a, b, c \in R$ If $a + c = b + c$, then $a = b$

Cancellation property w.r.t multiplication :- If $\frac{2}{4} \times 2 = \frac{1}{2} \times 2$, then $\frac{2}{4} = \frac{1}{2}$

$\forall\ a, b, c \in R$ If $ac = bc$, then $a = b$

Order properties

Trichotomy property:- $2=2$ or $2<10$ or $2>1$

$\forall\ a, b \in \mathbb{R}$ either $a=b$ or $a<b$ or $a>b$

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Transitive property $\forall\ a, b, c \in \mathbb{R}$

If $a>b \wedge b>c$ then $a>c$

$10>9 \wedge 9>8$ then $10>8$

If $a<b \wedge b<c$ then $a<c$

$1<5 \wedge 5<9$ then $1<9$

Additive property:- $\forall\ a, b, c \in \mathbb{R}$

If $a>b$, then $a+c > b+c$

If $10>5$, then $10+1 > 5+1$

$11>6$

If $a < b$, then $a + c < b + c$

If $5 < 10$, then $5 + 1 < 10 + 2$

$6 < 11$

Multiplicative property:- $\forall\ a, b, c \in \mathbb{R}$

If $a > b$, then $ac > bc$ if $c > 0$

$5 > 4$, then $(5)(2) > 4(2)$

$10 > 8$

If $a < b$, then $ac < bc$ if $c > 0$

$9 < 10$, then $(9)(2) < (10)(2)$

$18 < 20$

If $a > b$, then $ac < bc$ if $c < 0$

If $4 > 3$, then $4(-1) < 3(-1)$

$-4 < -3$ ↵ $-4 < -3$

True

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If $a < b$, then $ac > bc$ if $c < 0$

If $3 < 4$ then $(3)(-1) > (4)(-1)$

$-3 > -4$

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Division properties

$\forall\ a, b, c \in \mathbb{R}$

If $a < b$ then $\frac{a}{c} < \frac{b}{c}$ if $c > 0$

If $1 < 2$ then $\frac{1}{2} < \frac{2}{2} \Rightarrow 0.5 < 1$

If $a > b$ then $\frac{a}{c} > \frac{b}{c}$ if $c > 0$

If $2 > 1$ then $\frac{2}{2} > \frac{1}{2} \Rightarrow 1 > 0.5$

If $a < b$ then $\frac{a}{c} > \frac{b}{c}$ if $c < 0$

If $1 < 2$ then $\frac{1}{-2} > \frac{2}{-2} \Rightarrow -0.5 > -1$

If $a > b$, then $\frac{a}{c} < \frac{b}{c}$ if $c < 0$

If $2 > 1$, then $\frac{2}{-2} < \frac{1}{-2} \Rightarrow -1 < -0.5$

Reciprocal property: $\forall\ a, b \in \mathbb{R}$ (Both have same sign)

If $a > b$ then $\frac{1}{a} < \frac{1}{b}$

If $2 > 1$ then $\frac{1}{2} < \frac{1}{1}$ i.e $\frac{1}{2} < 1$

If $a < b$ then $\frac{1}{a} > \frac{1}{b}$

If $1 < 2$ then $\frac{1}{1} > \frac{1}{2}$ i.e. $1 > \frac{1}{2}$

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Example 5:- If $a = \frac{2}{3}, b = \frac{3}{2}, c = \frac{5}{3}$ then verify distributive properties over addition.

Left distributive law over addition

$$a(b+c) = ab + ac$$

L.H.S $= a(b+c)$

$= \frac{2}{3}\left(\frac{3}{2} + \frac{5}{3}\right)$

$= \frac{2}{3}\left(\frac{9+10}{6}\right)$

$= \frac{2}{3}\left(\frac{19}{6}\right)$

$= \frac{19}{9}$

R.H.S $= ab + ac$

$= \left(\frac{2}{3}\right)\left(\frac{3}{2}\right) + \left(\frac{2}{3}\right)\left(\frac{5}{3}\right)$

$= \frac{1}{1} + \frac{10}{9}$

$= \frac{9+10}{9}$

$= \frac{19}{9}$

L.H.S = R.H.S.

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Right distributive law of multiplication over addition

$$(a+b)c = ac + bc$$

L.H.S $= (a+b)c$

$= \left(\frac{2}{3} + \frac{3}{2}\right)\frac{5}{3}$

$= \left(\frac{4+9}{6}\right)\frac{5}{3}$

$= \left(\frac{13}{6}\right)\frac{5}{3}$

$= \frac{65}{18}$

R.H.S $= ac + bc$

$= \left(\frac{2}{3}\right)\left(\frac{5}{3}\right) + \left(\frac{3}{2}\right)\left(\frac{5}{3}\right)$

$= \frac{10}{9} + \frac{5}{2}$

$= \frac{20+45}{18}$

$= \frac{65}{18}$

L.H.S = R.H.S.

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Example:-6 Identify the property that justifies the statement

i):- If $a > 13$, then

$a + 2 > 15$

$a > 13$

$a + 2 > 13 + 2$

$a + 2 > 15$

Additive property

If $a > b$

$\Rightarrow a + c > b + c$

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ii):- If $3 < 9$ and $6 < 12$ then

$9 < 21$

$3 < 9$

$3 + 3 < 9 + 3$

$6 < 12$

Additive property

$3 + 6 < 9 + 12$

$9 < 21$

Additive property

(iii):- If $7 > 4$ and $5 > 3$ then

$35 > 12$

$7 > 4$

$5 > 3$

$35 > 12$

Multiplicative property

(iv):- If $-5 < -4$

$\Rightarrow 20 > 16$

$-5 < -4$

$-5(-4) > -4(-4)$

$20 > 16$

Multiplicative property

$\forall\ a, b \in \mathbb{R}$

If $a < b$

$ac > bc$ if $c < 0$

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